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Stabilization of the Burnett Equations and Application to Hypersonic Flows

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Numerical solutions of the Burnett equations for hypersonic flow at high altitudes in the continuum transitional regime were not possible except for some one-dimensional flows. It is shown from both analytical investigation and numerical computations that the Burnett equations are unstable to disturbances of small wavelengths. This fundamental instability arises in numerical computations when the grid spacing is less than the order of a mean free path and precludes Burnett flowfield computations above a certain maximum altitude for any given aerospace vehicle. A new set of equations termed the "augmented Burnett equations" has been developed and shown to be stable both by a linearized stability analysis and by direct numerical computations for one-dimensional and plane-two-dimensional flows. The latter represents the first known Burnett solutions for two-dimensional hypersonic flow over blunt leading edges. The comparison of these solutions with the conventional Navier-Stokes solutions reveals that the difference is small in low-altitude low-speed flows but significant in high-altitude hypersonic flows.

Nomenclature

c_v	= specific heat at constant volume
e	= total energy per unit volume
F, G	= flux vectors in Cartesian coordinates
Kn	= Knudsen number
L_0	= characteristic length in the stability analysis, $\mu_0/(\rho_0\sqrt{RT_0})$
M_∞	= freestream Mach number
Pr	= Prandtl number
p	= pressure
q_i	= heat transfer rate
R	= gas constant
T	= temperature
T'	= nondimensional temperature, $(T-T_0)/T_0$
T_s	= temperature of gas on wall surface
T_w	= wall temperature
t'	= nondimensional time, $t/(\mu_0/p_0)$
U	= vector of conserved variables
u, v	= velocity components in x and v directions
u_i	= components of velocity vector
u_s	= slip velocity of gas on wall surface
u'	= nondimensional velocity, $u/\sqrt{RT_0}$
x'	= nondimensional spatial coordinate, x/L_0
$\bar{\alpha}$	= accommodation coefficient on wall surface
γ	= ratio of specific heats
$\theta_1,\ldots,\theta_5$	= coefficients in the Burnett second-order heat transfer terms
θ_6, θ_7, ω_7	= coefficients in the augmented Burnett equations
κ	= heat conductivity coefficient
λ	= mean free path
μ	= viscosity coefficient
ρ	= density
ρ'	= nondimensional density, $(\rho-\rho_0)/\rho_0$
σ_{ij}	= viscous stress tensor
$\bar{\sigma}$	= reflection coefficient on wall surface
ω	= nondimensional circular frequency
$\omega_1,\ldots,\omega_6$	= coefficients in the Burnett second-order stress terms

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Introduction

FUTURE hypersonic vehicles, such as the National Aerospace Plane (NASP), are anticipated to involve operations at altitudes where the flow around the vehicles belongs to the continuum transitional regime and the thickness of the strong bow shock waves is a sizable or dominant part of the shock detachment distance. For flow under these conditions, computational fluid dynamics (CFD) is the main predicting tool that includes two approaches: the particulate-flow computation approach, such as the direct simulation Monte Carlo (DSMC) method of Bird[1] and the particle simulation method of Baganoff and McDonald,[2,3] and the continuum approach based on conservation equations.

Compared with the continuum approach, the DSMC approach in principle can simulate all of the correct physics of the flow. However, the approach can require a relatively large amount of computer time, especially at lower altitudes. For continuum transitional flow, the advantage of the continuum CFD is its computational efficiency compared with DSMC computations. However, the conventional governing equations of the continuum approach, the Navier-Stokes equations, are appropriate only for flow in the continuum regime and become inaccurate as the flow enters the continuum transitional regime. Therefore, we need to develop some other set of constitutive equations, more advanced than the Navier-Stokes equations, to provide realistic continuum-flow computations for hypersonic flows at these high altitudes.

To extend the continuum approach beyond the Navier-Stokes level, Fiscko and Chapman[4,5] reinvestigated the Burnett equations,[6] which are one higher order approximation to the Boltzmann equation than the Navier-Stokes equations. They found that the Burnett equations provide much greater accuracy than the Navier-Stokes equations for one-dimensional shock wave structure in monatomic gases. However, numerical solutions of the Burnett equations for hypersonic flow in the continuum transitional regime have not been possible for flow with very fine grids because the Burnett equations are unstable to disturbances of small wavelengths. This fundamental instability arises in numerical computations when the grid spacing is less than the order of a mean free path and makes it impossible to use these equations to compute flows in two and three dimensions above a certain altitude for any vehicle.

The objectives of this paper are to overcome the instability of the Burnett equations and to obtain numerical solutions of the stabilized Burnett equations for practical flow problems afterwards. Guided by a linearized stability analysis, this paper develops a new set of equations termed the "augmented Burnett equations" to stabilize the conventional Burnett equations. This stability involves both analytical stability of the linearized Burnett equations and numerical stability in flowfield computations. Numerical solutions of the augmented Burnett equations are obtained for normal shock waves and for two-dimensional hypersonic flows over blunt bodies. Some of these results are compared with existing experimental data and DSMC results.

Governing Equations

Translational nonequilibrium of rarefied gas flow can be characterized by the Knudsen number Kn that is the ratio of a molecular mean free path to the macroscopic characteristic length of the flow. When Kn increases from 0 through 1 to ∞, the gas flow regime changes from the translational equilibrium regime (continuum regime) through the continuum transitional regime to the free-molecule regime. The continuum transitional regime in this paper refers to the flow of Kn less than the order of 1 but not negligibly small.

The Boltzmann equation, which describes the evolution of the molecular distribution function, is the general governing equation for a monatomic perfect gas flow from the continuum to free-molecule flow regime. However, numerical solutions of the Boltzmann equation are so computationally intensive to obtain that they are limited to rarefied flow of large Kn at high altitudes. For flow at small Knudsen numbers near the continuum limit, it is more efficient to use the continuum assumption to study the flow from a macroscopic point of view.

In the continuum approach, the macroscopic governing equations are the conservation equations of mass, momentum, and energy as follows (in two dimensions):

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 \tag{1}$$

where

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix} \tag{2}$$

$$F = \begin{bmatrix} \rho u \\ \rho u^2 + p + \sigma_{11} \\ \rho u v + \sigma_{12} \\ (e + p + \sigma_{11})u + \sigma_{12} v + q_1 \end{bmatrix} \tag{3}$$

$$G = \begin{bmatrix} \rho v \\ \rho u v + \sigma_{21} \\ \rho v^2 + p + \sigma_{22} \\ (e + p + \sigma_{22})v + \sigma_{21} u + q_2 \end{bmatrix} \tag{4}$$

$$p = \rho R T \tag{5}$$

$$e = \rho\left(c_v T + \frac{u^2 + v^2}{2}\right) \tag{6}$$

It can be shown that the macroscopic conservation equations, Eq. (1), are the lower order moment equations of the Boltzmann equation and are generally valid for flow at any Knudsen number. These equations, however, do not form a complete set of equations. The terms σ_{ij} and q_i in Eqs. (3) and (4) are the results of translational nonequilibrium effect. Additional equations relating the σ_{ij} and q_i with gradients of macroscopic flow variables, i.e., the constitutive equations, are needed to close Eq. (1).

It is the constitutive relations that introduce approximations to the general conservation equations. The constitutive relations for gas at low altitudes under normal pressure are different from those at high altitudes under low pressure.

For flow at low altitudes where $Kn \ll 1$, the constitutive relations are the linear Stokes law for stress terms and the linear Fourier law for heat flux terms. The conservation equations, Eq. (1), together with these linear constitutive relations form the Navier-Stokes equations that are the governing equations in studying viscous flow at low altitudes in the continuum limit.

As the altitudes increase, however, the Knudsen number increases and the gas becomes more and more nonequilibrium. As a result, the linear constitutive relations in the Navier-Stokes equations become inaccurate, and nonlinear and higher order terms are needed in the constitutive relations. The second-order (with respect to Kn) constitutive relations can be derived by the Chapman-Enskog method in solving the Boltzmann equation.[6,7] Their general expressions are

$$\begin{aligned} \sigma_{ij}^{(B)} = & -2\mu \overline{\frac{\partial u_i}{\partial x_j}} + \frac{\mu^2}{p}\Bigg\{\omega_1 \frac{\partial u_k}{\partial x_k}\overline{\frac{\partial u_i}{\partial x_j}} \\ & + \omega_2\left[-\overline{\frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\right)} - \overline{\frac{\partial u_k}{\partial x_i}\frac{\partial u_j}{\partial x_k}} - 2\overline{\overline{\frac{\partial u_i}{\partial x_k}}\frac{\partial u_k}{\partial x_j}}\right] \\ & + \omega_3 R\overline{\frac{\partial^2 T}{\partial x_i \partial x_j}} + \omega_4 \overline{\frac{1}{\rho T}\frac{\partial p}{\partial x_i}\frac{\partial T}{\partial x_j}} \\ & + \omega_5 \overline{\frac{R}{T}\frac{\partial T}{\partial x_i}\frac{\partial T}{\partial x_j}} + \omega_6 \overline{\overline{\frac{\partial u_i}{\partial x_k}}\,\overline{\frac{\partial u_k}{\partial x_j}}}\Bigg\} \end{aligned} \tag{7}$$

and

$$\begin{aligned} q_i^{(B)} = & -\kappa \frac{\partial T}{\partial x_i} + \frac{\mu^2}{\rho}\Bigg\{\theta_1 \frac{1}{T}\frac{\partial u_k}{\partial x_k}\frac{\partial T}{\partial x_i} \\ & + \theta_2 \frac{1}{T}\left[\frac{2}{3}\frac{\partial}{\partial x_i}\left(T\frac{\partial u_k}{\partial x_k}\right) + 2\frac{\partial u_k}{\partial x_i}\frac{\partial T}{\partial x_k}\right] \\ & + \theta_3 \frac{1}{\rho}\frac{\partial p}{\partial x_k}\overline{\frac{\partial u_k}{\partial x_i}} + \theta_4 \frac{\partial}{\partial x_k}\left(\overline{\frac{\partial u_k}{\partial x_i}}\right) \\ & + \theta_5 \frac{1}{T}\frac{\partial T}{\partial x_k}\overline{\frac{\partial u_k}{\partial x_i}}\Bigg\} \end{aligned} \tag{8}$$

where a bar over a derivative designates a nondivergent symmetrical tensor,

$$\overline{A_{ij}} = \begin{cases} \dfrac{A_{ij} + A_{ji}}{2} - \dfrac{1}{3}(A_{11} + A_{22} + A_{33}) & \text{if } i = j \\ \dfrac{A_{ij} + A_{ji}}{2} & \text{if } i \neq j \end{cases}$$

In Eqs. (7) and (8), the coefficients ω_i and θ_i can be computed by using the Chapman-Enskog expansion with a molecular repulsive force model. So far, only the coefficients for the two extreme cases—the hard-sphere gas model and the Maxwellian gas model ($F \propto r^{-5}$)—have been computed to a high-order accuracy as shown in Table 1.

Table 1 Values of the Burnett coefficients

	Maxwell molecules	Hard-sphere molecules
ω_1	10/3	4.056
ω_2	2	2.028
ω_3	3	2.418
ω_4	0	0.681
ω_5	3	0.219
ω_6	8	7.424
θ_1	75/8	11.644
θ_2	−45/8	−5.822
θ_3	−3	−3.090
θ_4	3	2.418
θ_5	117/4	25.157

Because the molecular model for a real gas falls in between the two extreme cases shown in Table 1, Lumpkin[8] suggested using values of ω_i and θ_i for real gases interpolated linearly in the temperature-viscosity exponent ω from the data in Table 1. This interpolation is first-order accurate.

The conservation equations, Eq. (1), together with the second-order constitutive relations given by Eqs. (7) and (8) are termed the Burnett equations. The Burnett equations are used in this paper to improve the accuracy of the Navier-Stokes equations for high-altitude hypersonic flows in the continuum transitional regime.

It is noted that for flow in the free molecule regime ($Kn \gg 1$), which is out of the application limit of the continuum approach of the present paper, the Burnett equations are no longer appropriate and the flow can no longer be described by the continuum approach because no constitutive relations are available to close the conservation equations. Such flows can be studied by numerical solutions of the Boltzmann equation or by the DSMC techniques.

Difficulties Associated with the Burnett Equations

Although the Burnett equations seem to be a natural extension of the Navier-Stokes equations in the continuum transitional regime, the Burnett equations have never been used successfully in practical applications. The difficulties associated with the Burnett equations, as pointed out by many authors,[9-12] are as follows.

Theoretical Difficulties

There are several theoretical difficulties associated with the Burnett equations. First, the Chapman-Enskog series that leads to the Burnett equations is asymptotic in the Knudsen number Kn, and the convergence properties of the series are not known. Second, there is no theoretical proof that the Burnett equations always have a positive dissipation function to satisfy the second law of thermodynamics. Third, the Burnett equations are found to be frame dependent in rotating coordinates. Fourth, the Burnett equations increase the order of the Navier-Stokes equations by one and hence are expected to require more boundary conditions, which are currently unknown. The uncertainty on the boundary conditions also raises the question about the uniqueness of the solutions of the Burnett equations for boundary value problems. Fifth, there are questions about the conventional approximation treatment of the substantial derivatives in the Burnett stress and heat flux terms and uncertainty about whether slightly different forms of the Burnett equations, such as those derived by Woods from mean free path consideration, may be preferable to the conventional form derived from the Chapman-Enskog expansion of the Boltzmann equations.[13,14] All of these questions concerning the Burnett equations need to be answered. Because of these difficulties, the Burnett equations have been considered by many to be useless in fluid mechanics until Fiscko and Chapman reinvestigated them for possible applications in hypersonic flow computations.

Difficulties in Obtaining Numerical Solutions

Although some of the theoretical questions just cited are currently under investigation, the more important issue from the engineering point of view is whether the Burnett equations provide better physics than the Navier-Stokes equations as the constitutive relations of gas flow. This issue can be investigated numerically by solving the Burnett equations for continuum transitional flow. The merit of the Burnett equations can be evaluated by comparing their numerical solutions with experimental results or results from other predicting techniques such as the DSMC technique.

Many attempts have been made to solve the Burnett equations for flow structure within normal shock waves. Sherman and Talbot[15] integrated the Burnett equations for weak shock waves of $M \simeq 1.9$; Simon and Foch[16] obtained the Burnett shock solutions for $M = 4$. Fiscko and Chapman[5] obtained Burnett solutions for shock waves of some gas models of Mach number up to 50 and compared the results of the Burnett equations with Navier-Stokes, DSMC, and available experimental results. The results of Fiscko and Chapman show that the Burnett equations give a significant improvement on accuracy over the Navier-Stokes equations. These results show that the Burnett equations do improve the results of the Navier-Stokes equations when the flow belongs to the continuum transitional regime.

However, because of an inherent instability of the Burnett equations, no numerical solutions of these equations could be obtained for hypersonic flow until the work of Fiscko and Chapman. Even in Fiscko and Chapman's computations, the computations became unstable for strong shock waves or when fine computational mesh was used. This instability makes it impossible to apply these equations to practical flow problems and also raises the question that the Burnett equations may not be well posed.

The instability of the Burnett equations was analyzed by Bobylev[17] using a linearized stability analysis. Bobylev showed that the solutions of the Burnett equations are exponentially unstable to periodic perturbations when the wavelengths of the perturbations are shorter than some critical length of the order of a mean free path.

A linearized stability analysis to the one-dimensional Burnett equations is considered by assuming that the solutions of the Burnett equations are periodic in the x direction as follows:

$$V' = V_0' \exp[\phi t' + i\omega x'] \tag{9}$$

where $(\,)'$ represents a nondimensional variable, $V' = (\rho, u, T)'$, and ω has the following relation with the wavelength L: $\omega = 4.92\lambda/L$. The condition for the Burnett solutions to be stable is

$$R(\phi) \le 0 \tag{10}$$

where $R(\phi)$ represents the real part of the complex number ϕ.

Substituting Eq. (9) into the linearized Burnett equations results in the characteristic equation in the following form:

$$\phi = f(\omega) = f\left(4.92\,\frac{\lambda}{L}\right) \tag{11}$$

where λ is the dimensionless parameter defined by

$$\lambda = \frac{16}{5}\,\frac{\mu}{\rho\sqrt{2\pi RT}} \tag{12}$$

This is the same as the Maxwell mean free path for a hard sphere gas.

Figure 1 shows the trajectories of ϕ ($\phi = \alpha + i\beta$) in the complex plane as L decreases from $+\infty$ to 0 (Kn increases from 0 to $+\infty$). The figure shows that the trajectories of the Navier-Stokes equations are always in the stable region ($\alpha \le 0$) and move toward the negative α direction as L decreases. This

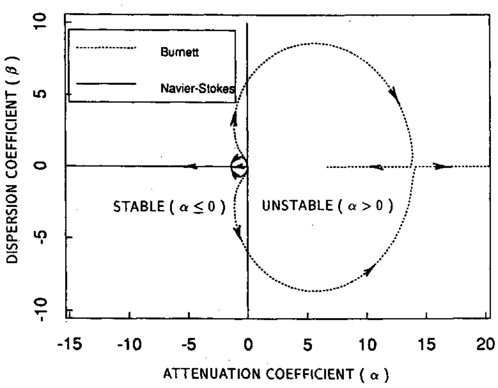

Fig. 1 Maxwellian gas characteristic trajectories for flows in both one and two dimensions. The arrows show the direction in decreasing wavelength.

means that the Navier-Stokes equations are always stable for the linearized analysis. On the other hand, two branches of the trajectories of the Burnett equations go into the unstable region when $L \leq L_{cr}$ ($L_{cr} = 2.04\lambda$ for the Maxwellian gas), i.e, if the wavelength is smaller than L_{cr}, the Burnett equations become unstable. This instability explains the difficulties in solving the Burnett equations, i.e., short wavelength numerical disturbances corresponding to a fine computational grid always make the computation unstable.

Compared with the Navier-Stokes equations, the Burnett equations are more accurate in the transition regime ($Kn < 1$). On the other hand, when $Kn > 1$ where both the Navier-Stokes and Burnett equations are not expected to apply, the Navier-Stokes equations are always well posed whereas the Burnett equations are ill posed or unstable. As a result, the numerical solutions of the Navier-Stokes equations are easily available, but those of the Burnett equations are very difficult to obtain because the Burnett equations are not stable to perturbations corresponding to fine grid spacing.

Augmented Burnett Equations

According to the preceding section, the Burnett equations improve the accuracy of the Navier-Stokes equations in the transitional regime but are ill posed when $Kn > 1$. The main goal of this paper is to stabilize the Burnett equations so that the new stabilized equations not only maintain the accuracy of the conventional second-order Burnett equations in the transitional regime ($Kn < 1$) but also are stable in the linearized stability analysis at any Knudsen number.

Because only second-order accuracy is needed in the constitutive relations in the present research, we choose to stabilize the Burnett equations by augmenting the Burnett stress and heat flux terms with some higher-order terms. The Burnett equations plus the augmented terms form the new constitutive equations termed "the augmented Burnett equations" as follows:

$$\begin{cases} \sigma_{ij} = \sigma_{ij}^{(B)} + \sigma_{ij}^{(a)} \\ q_i = q_i^{(B)} + q_i^{(a)} \end{cases} \tag{13}$$

where $\sigma_{ij}^{(B)}$ and $q_i^{(B)}$ are given by Eqs. (7) and (8). The augmented terms $\sigma_{ij}^{(a)}$ and $q_i^{(a)}$ are chosen to stabilize the conventional Burnett equations.

To maintain the second-order accuracy, we can only add the third or higher order terms to stabilize the conventional Burnett equations. The most natural choice would be the complete super Burnett equations. However, it can be shown that the super Burnett equations, like the conventional Burnett equations, are also unstable to small perturbations when the wavelengths are smaller than some critical value. Therefore, we augment the conventional Burnett equations with some terms of third-order derivatives that are picked from the super Burnett equations but have different coefficients to ensure both analytical and numerical stability. The choice of stabilization terms is a practical one but not unique. The so-chosen one-dimensional $\sigma_{11}^{(a)}$ and $q_1^{(a)}$ are:

$$\sigma_{11}^{(a)} = \frac{\mu^3}{p^2}(\omega_7 R T u_{xxx}) \tag{14}$$

$$q_1^{(a)} = \frac{\mu^3}{p\rho}\left(\theta_7 R T_{xxx} + \theta_6 \frac{RT}{\rho} \rho_{xxx}\right) \tag{15}$$

where the coefficients ω_7, θ_6, and θ_7 of the augmented terms are selected so that the new augmented Burnett equations are stable by linearized analysis. The coefficients are chosen to be those of Wang Chang,[18] for a Maxwellian gas: $\omega_7 = 2/9$, $\theta_6 = -5/8$, and $\theta_7 = 11/16$.

We have generalized the one-dimensional augmented terms in Eqs. (14) and (15) to the following general expressions:

$$\sigma_{ij}^{(a)} = \frac{\mu^3}{p^2}\left[\frac{3}{2}\omega_7 \, RT \overline{\frac{\partial}{\partial x_j}\left(\frac{\partial^2 u_i}{\partial x_k \partial x_k}\right)}\right] \tag{16}$$

$$q_i^{(a)} = \frac{\mu^3}{p\rho}\left[\theta_7 R \frac{\partial}{\partial x_i}\left(\frac{\partial^2 T}{\partial x_k \partial x_k}\right) + \theta_6 \frac{RT}{\rho}\frac{\partial}{\partial x_i}\left(\frac{\partial^2 \rho}{\partial x_k \partial x_k}\right)\right] \tag{17}$$

Figure 2 shows the characteristic trajectories for both the one-dimensional and the two-dimensional augmented Burnett equations. The trajectories of the new equations are always in the stable region. Therefore, the augmented Burnett equations do stabilize the conventional Burnett equations by the linearized analysis. It is noted that the stability analysis so far has been for the linearized Burnett equations only. The nonlinear terms in the Burnett equations that are important in most of the practical hypersonic applications are not included in the stability analysis. Therefore the linearized stability analysis provides only the necessary stability condition for the nonlinear Burnett equations. Whether the nonlinear augmented Burnett equations are stable in practical computations needs to be tested. In the following section, we will test the stability properties of both the conventional Burnett equations and the augmented Burnett equations for practical numerical computations by progressively refining the computational meshes using a stable numerical method.

Numerical Solutions of Augmented Burnett Equations

Numerical solutions of the new augmented Burnett equations are obtained for both one-dimensional and two-dimensional hypersonic flows. The cases include one-dimensional hypersonic normal shock waves and two-dimensional hypersonic flows past blunt bodies. The latter cases represent the first known Burnett solutions for hypersonic flows past blunt bodies.

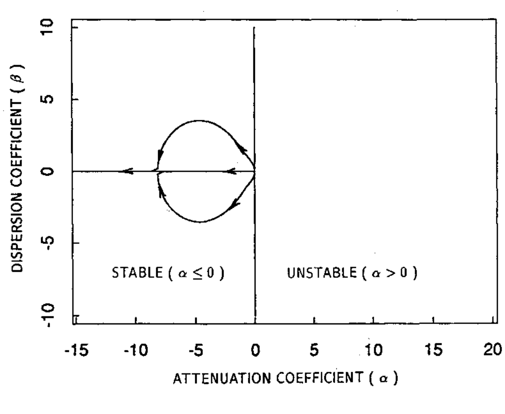

Fig. 2 Maxwellian gas characteristic trajectories for the augmented Burnett equations in both one and two dimensions. The arrows show the direction in decreasing wavelength.

The solutions of the one-dimensional augmented Burnett solutions are compared with the available solutions of the one-dimensional conventional Burnett results obtained by Fiscko and Chapman. The Burnett solutions are also compared with particle simulation results and experimental results. The computations show that the augmented Burnett solutions agree with the conventional Burnett results when $Kn < 1$ and stabilize the conventional Burnett equations when Kn is much larger than one. The numerical computations also show that it is necessary to use the augmented Burnett equations to obtain stable numerical solutions.

The implicit line Gauss-Seidel iteration method for the Navier-Stokes equations[19] is used to solve the Burnett equations. The inviscid terms in the conservation equations are computed by using the flux-splitting method, and all of the viscous terms, including the Burnett terms, are computed by using central difference approximations.

Since the issue of the surface boundary conditions for the Burnett equations has not been resolved, two-dimensional hypersonic flows past blunt bodies are chosen to be our test cases. For these test cases, the Burnett terms are important mainly in the nonequilibrium region within the bow shock waves, which are away from the wall. In the boundary layers, the flow is almost in equilibrium, and the Navier-Stokes equations are accurate and the higher order Burnett terms are negligible. Hence the higher derivative terms play a negligible role in solving the flow equations near the wall, and the solutions of the Burnett and Navier-Stokes equations are almost identical near the wall. The Maxwell/Smoluchowski slip boundary conditions, which are first-order accurate, are used for the Burnett equations in the present computations (in Cartesian coordinates):

$$u_s = \frac{2-\bar{\sigma}}{\bar{\sigma}} \bar{l} \left(\frac{\partial u}{\partial y}\right)_s + \frac{3}{4}\frac{\mu}{\rho T}\left(\frac{\partial T}{\partial x}\right)_s \tag{18}$$

and

$$T_s = T_w + \frac{2-\bar{\alpha}}{\bar{\alpha}} \frac{2\gamma}{\gamma+1} \frac{\bar{l}}{Pr}\left(\frac{\partial T}{\partial y}\right)_s \tag{19}$$

where

$$\bar{l} = \frac{2\mu}{\rho}\sqrt{\frac{\pi}{8RT}}$$

The subscript s represents the flow variables on the wall surface. For this study, complete accommodation is assumed, i.e., $\bar{\sigma} = 1$ and $\bar{\alpha} = 1$.

Because of space limitation, only the main results of the Burnett equations are presented in this paper. Additional results can be found in the complete paper.[20]

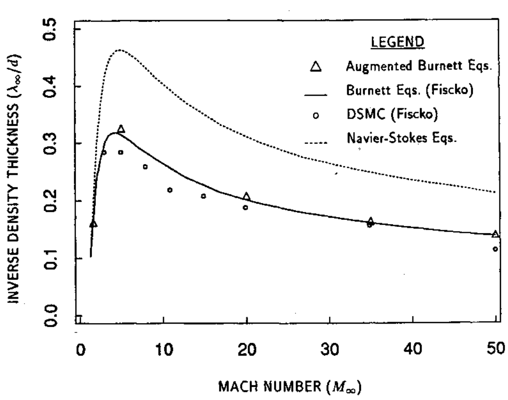

Fig. 3 Argon shock wave inverse density thickness.

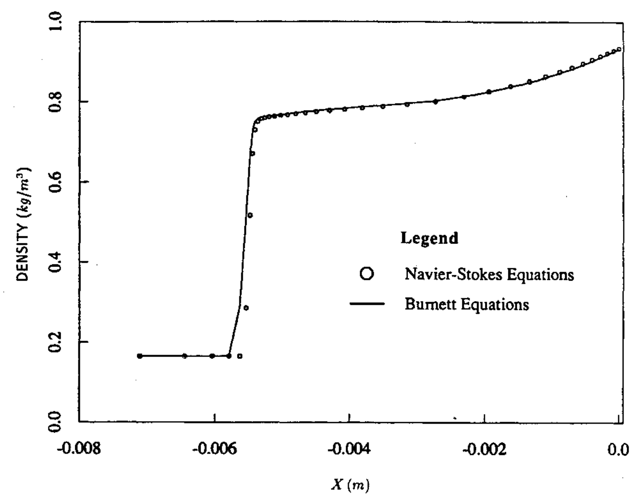

Fig. 4 Density along stagnation streamline for case I: $M_\infty = 4.0$, and $Kn_\infty = 0.67 \times 10^{-4}$.

Hypersonic Shock Wave Structure

We have computed the hypersonic shock wave structure in a monatomic gas with the molecular weight of argon by using the conventional Burnett equations and the augmented Burnett equations to test the stability of both equations and to compare their results with those of the conventional Burnett equations obtained by Fiscko and Chapman.

We tested the stability of the conventional Burnett equations and the augmented Burnett equations by progressively refining the computation mesh (increasing the number of grid points within a fixed length). The computations were made for Mach 20 shock waves in Maxwellian gas, since this gas model had exhibited the most severe instabilities in Fiscko's work. We started the computations for the shock structure from a 20-grid-point coarse mesh spanning a computational domain of 100 mean free paths. The computations for both the conventional Burnett equations and the augmented Burnett equations were stable for this coarse mesh. Shock profiles were obtained for both equations. Then we computed the same shock structure with more grid points spanning the same length. The conventional Burnett equations became unstable when the number of grid points exceeded 87 for this case. This critical grid point spacing corresponds to $\Delta x/\lambda = 0.83$. In contrast, the computations of the augmented Burnett equations were stable for all of the numbers of grid points tested (up to 6000 grid points, the largest number tested, which corresponds to $\Delta x/\lambda < 0.01$).

The stability test shows that the conventional Burnett equations are unstable to short wavelength perturbations. On the other hand, the augmented Burnett equations stabilize the Burnett equations in the numerical computations for shock wave structure.

We next computed the argon shock wave structure for Mach numbers ranging from 1.59 to 50.0 disregarding the effect of ionization, with $Pr = 2/3$, $\gamma = 5/3$, and $\mu \propto T^{0.72}$ for the monatomic gas. Figure 3 shows the inverse density thickness of the shock waves. The inverse density thickness of the augmented Burnett equations agrees well with Fiscko's conventional Burnett results. As pointed out by Fiscko, the results of the Burnett equations are always closer to DSMC than the Navier-Stokes equations. The results show that the augmented Burnett equations maintain the second-order accuracy of the conventional Burnett equations for computing shock wave structure. Besides the shock profiles of the augmented Burnett equations for argon presented in this paper, Lumpkin and Chapman[21] recently have used the augmented Burnett equations to compute the shock structure in hard-sphere and Maxwellian gases for Mach numbers up to 50. They obtained similar results for the augmented Burnett equations.

Hypersonic Flow Past Two-Dimensional Cylinders

We have extended the computations for the one-dimensional shock wave structure to plane-two-dimensional flows. The first problem is supersonic and hypersonic flows past cylindrical leading edges. Freestream conditions of the two-dimensional hypersonic flows past cylindrical leading edges are the standard atmosphere at various altitudes ranging from continuum to transitional regime, with $Pr = 0.72$, $\gamma = 1.4$, and the Sutherland viscosity law.

Figure 4 shows the density distributions along the stagnation streamline for flow in the continuum regime. The flow conditions are $M_\infty = 4.0$ and $Kn_\infty = 0.67 \times 10^{-4}$, where the Knudsen number is based on freestream conditions and the radius of the cylinder. Since $Kn_\infty \ll 1.0$, the flow belongs to the continuum regime. Therefore it can be accurately described by the Navier-Stokes equations (except in the thin region within the bow shock wave). The figure shows that the Burnett equations give essentially the same results as the Navier-Stokes equations at this low altitude case.

Figure 5 shows the temperature distributions along the stagnation streamline for the flow at a larger Knudsen number. The flow conditions are altitude = 75 km, $M_\infty = 10.0$, $Kn_\infty = 0.102$, and $T_{\text{wall}} = 1000.0$ K. Since the Knudsen number increases from 0.67×10^{-4} of the last case to 0.102 of the present case, the Navier-Stokes equations begin to be inaccurate. The figure shows that the difference between the Burnett equations and Navier-Stokes equations is evident and appears mainly in the bow shock away from the wall. Meanwhile, the results of the augmented Burnett equations agree well with the conventional Burnett equations. Hence, the augmented Burnett equations maintain the accuracy of the conventional Burnett equations when the solutions of the latter equations are possible.

Figures 6 and 7 show the temperature contours for the Navier-Stokes equations and the augmented Burnett equations. The figures show that the two-dimensional augmented Burnett equations result in a thicker bow shock wave.

Figure 8 shows temperature distribution along the stagnation streamline for the flow at a still larger Knudsen number. The flow conditions are the same as in the preceding case except altitude = 90 km and $Kn_\infty = 1.185$. The flow in this case is well into the continuum transitional regime. The difference between the augmented Burnett equations and the Navier-Stokes equations becomes significant in this case. Meanwhile, the computations for the augmented Burnett equations are stable, whereas the computations for conventional Burnett equations are unstable. These results show that the augmented Burnett equations stabilize the conventional Burnett equations in two dimensions, and the difference between the Burnett and Navier-Stokes solutions is negligible at low altitudes but becomes larger as altitudes increase, mainly within the bow shock waves.

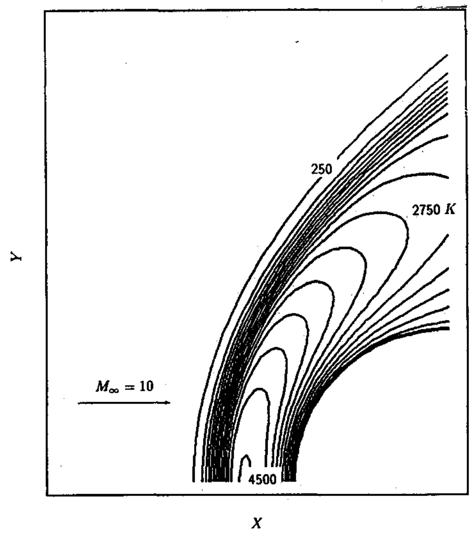

Fig. 6 Navier-Stokes temperature contours for case II: $M_\infty = 10$, $Kn_\infty = 0.1$, and altitude = 75 km.

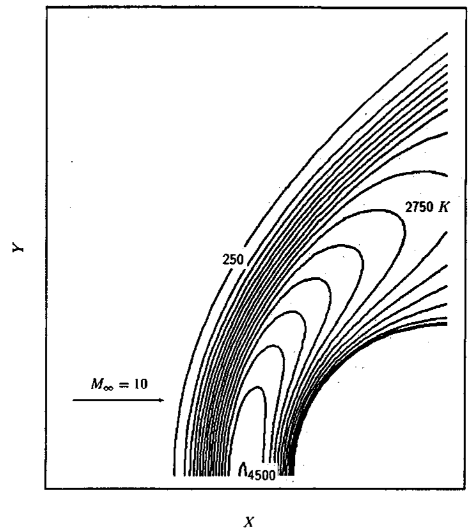

Fig. 7 Augmented Burnett temperature contours for case II: $M_\infty = 10$, $Kn_\infty = 0.1$, and altitude = 75 km.

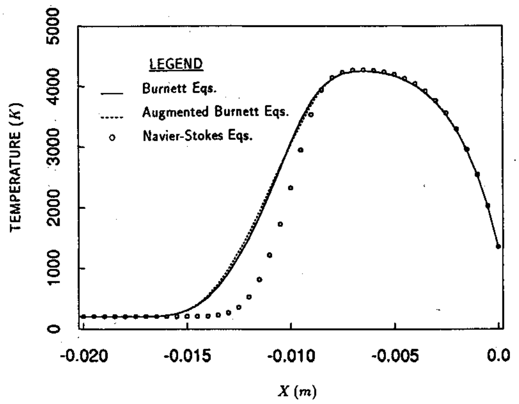

Fig. 5 Temperature along stagnation streamline for case II: $M_\infty = 10$, $Kn_\infty = 0.1$, and altitude = 75 km.

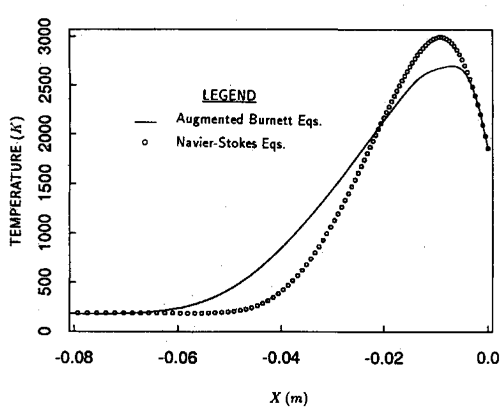

Fig. 8 Temperature along stagnation streamline for case III: $M_\infty = 10$, $Kn_\infty = 1.2$, and altitude = 90 km.

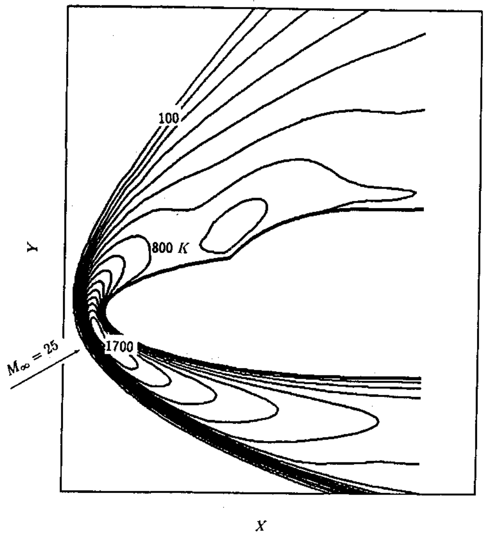

Fig. 9 Augmented Burnett translational temperature contours for case IV: $M_\infty = 25$, angle of attack = 30 deg, and $\lambda_\infty = 1.05 \times 10^{-3}$.

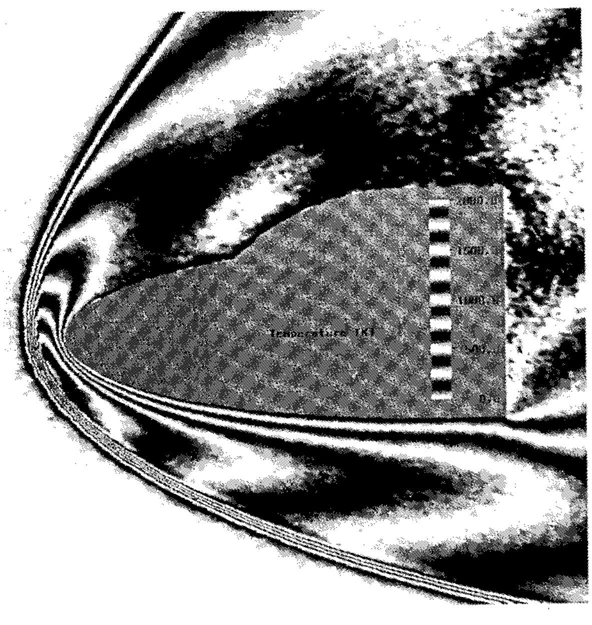

Fig. 10 Particle simulation translational temperature contours by Feiereisen: $M_\infty = 25$, angle of attack = 30 deg, and $\lambda_\infty = 1.05 \times 10^{-3}$.

It is noted that Tannehill and Eisler[22] solved the two-dimensional Burnett equations in 1976 for hypersonic flow over a flat plate with sharp leading edge, which was the first two-dimensional attempt with these equations. Their computations were necessarily restricted to a coarse mesh because of computer limitations at that time. The evenly spaced mesh sizes in their computations corresponded to $\Delta x/\lambda_\infty = 4.8$ and $\Delta y/\lambda_\infty = 1.9$, which are larger than the minimum value $\Delta x/\lambda_1 = 0.83$ for shock computation stability. Therefore, their computations may not have run into an instability problem because of the relatively coarse mesh used. It is also noted that they used different surface boundary conditions for the Burnett equations (Schamberg[23] boundary conditions) than for the Navier-Stokes equations (Maxwell/Smoluchowski[9] boundary conditions), and obtained results for the Burnett equations that appeared worse than for the Navier-Stokes equations. In contrast, one-dimensional hypersonic shock structure tests, which involve no uncertainty about boundary conditions, have shown the Burnett results to be much better than the Navier-Stokes results.

Hypersonic Flow Past a Double Ellipse

We have computed two-dimensional hypersonic airflow past a double ellipse in the transitional regime. The same problem was computed by Feiereisen[24] using the particle simulation method of Baganoff. In Feiereisen's computations, hard-sphere model and a rotational nonequilibrium model of collision number 2 were used. We use the same models to compute the flow by using the augmented Burnett equations and compare our results with Feiereisen's.

The flow conditions are: $M_\infty = 25.0$, $Kn_\infty = 0.28$, angle of attack = 30 deg, $T_\infty = 13.5$ K, and $T_w = 620.0$ K. Because of the low T_∞ and T_w, the vibrational mode of the air flow is assumed to be frozen and only translational and rotational nonequilibrium need to be considered for this case.

Although the Burnett equations have only been derived for the monatomic gas, Lumpkin[8] demonstrated that the Burnett equations plus a rotational nonequilibrium model provide satisfactory shock wave structure for a diatomic gas such as nitrogen. Therefore, we used the Burnett equations to model the translational nonequilibrium for the viscous stress and translational heat transfer and used first-order equations for the rotational heat transfer, with separate translational and rotational temperatures.

Figures 9 and 10 show the translational temperature contours of the flow. The translational temperature contours of the augmented Burnett equations and Feiereisen's results are plotted together for comparison. The figures show that translational temperature contours for the augmented Burnett equations agree well with the particle simulation results.

Conclusions

A new set of equations termed the augmented Burnett equations has been developed to overcome the instability of the conventional Burnett equations to disturbances of small wavelength. We have computed both one-dimensional shock wave structures and two-dimensional flows past blunt leading edges by using the augmented Burnett equations and compared the Burnett results with available experimental, DSMC, and Navier-Stokes results. The main conclusions are as follows:

1) The augmented Burnett equations are always stable in the linearized stability analysis as well as in both one- and two-dimensional numerical tests produced to date.

2) The augmented Burnett equations maintain the accuracy of the conventional Burnett equations in the continuum transitional regime.

3) Comparison of the Burnett solutions with the conventional Navier-Stokes solutions reveals that the difference is small in low-altitude low-speed flows but significant in high-altitude hypersonic flows.

4) Further research is needed to resolve many remaining theoretical questions about the applicability and limitation of the Chapman-Enskog-Burnett expansion solutions of the Boltzmann equations, such as the issue of boundary conditions for the high-order Burnett equations, and the uniqueness of the solutions of the Burnett equations for boundary value problems; the conventional approximation treatment of the substantial derivatives in the Burnett stress and heat flux terms; and whether slightly different forms of the Burnett equations, such as those derived by Woods from mean free path consideration, may be preferable to the conventional form derived from the Chapman-Enskog expansion of the Boltzmann equation.

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References

[1]Bird, G. A., "Monte Carlo Simulation of Gas Flows," *Annual Reviews of Fluid Mechanics,* Vol. 10, 1978, pp. 11–13.

[2]Baganoff, D., "Vectorization of a Particle Code Used in the Simulation of Rarefied Hypersonic Flow," Symposium on Computational Technology for Flight Vehicles, Washington, DC, Nov. 1990.

[3]Baganoff, D., and McDonald, J. D., "A Collision-Selection Rule for a Particle Simulation Method Suited to Vector Computers," *Physics of Fluids A,* Vol. 2, July 1990, pp. 1248–1259.

[4]Fiscko, K. A., "Study of Continuum Higher Order Closure Models Evaluated by a Statistical Theory of Shock Structure, Ph.D. Thesis, Stanford Univ., Stanford, CA, Aug. 1988.

[5]Fiscko, K. A., and Chapman, D. R., "Comparison of Burnett, Super-Burnett, and Monte Carlo Solutions for Hypersonic Shock Structure," *Rarefied Gas Dynamics,* edited by E. P. Muntz, D. P. Weaver, and D. H. Campbell, Vol. 118, Progress in Astronautics and Aeronautics, AIAA, Washington, DC, 1989.

[6]Chapman, S., and Cowling, T. G., *The Mathematical Theory of Non-Uniform Gases,* 3rd ed., Cambridge Univ. Press, Cambridge, England, UK, 1970.

[7]Wang Chang, C. S., and Uhlenbeck, G. E., "On the Transport Phenomena in Rarefied Gases," *Studies in Statistical Mechanics,* edited by J. De Boer and G. E. Uhlenback, Vol. V, American Elseuler Publishing Co., New York, 1970, pp. 1–17.

[8]Lumpkin, F. E., III, "Development and Evaluation of Continuum Models for Translational-Rotational Nonequilibrium," Ph.D. Thesis, Stanford Univ., Stanford, CA, March 1990.

[9]Schaaf, S. A., and Chambre, P. L., "Flow of Rarefied Gases," *Princeton Aeronautical Paperbacks,* Vol. 8, Princeton Univ. Press, Princeton, NJ, 1961.

[10]Kogan, M. N., *Rarefied Gas Dynamics,* Plenum Press, New York, 1969.

[11]Cercignani, C., *The Boltzmann Equation and Its Applications,* Springer-Verlag, New York, 1988.

[12]Woods, L. C., "Frame-Indifferent Kinetic Theory," *Journal of Fluid Mechanics,* Vol. 136, Nov. 1983, pp. 423–433.

[13]Woods, L. C., "Transport Processes in Dilute Gases over the Whole Range of Knudsen Number, Pt. I: General Theory," *Journal of Fluid Mechanics,* Vol. 93, No. 3, 1979, pp. 585–607.

[14]Woods, L. C., "On the Thermodynamics of Nonlinear Constitutive Relations in Gasdynamics," *Journal of Fluid Mechanics,* Vol. 101, No. 2, 1980, pp. 225–242.

[15]Sherman, F. S., and Talbot, L., "Experiment versus Kinetic Theory for Rarefied Gases," *Rarefied Gas Dynamics,* edited by F. M. Devienne, Pergamon, New York, 1960, pp. 161–191.

[16]Simon, C. E., and Foch, J. D., Jr., "Numerical Integration of the Burnett Equations for Shock Structure in a Maxwell Gas," *Rarefied Gas Dynamics,* edited by J. L. Potter, Progress in Astronautics and Aeronautics, Vol. 51, AIAA, New York, 1977.

[17]Bobylev, A. V., "The Chapman-Enskog and Grad Methods for Solving the Boltzmann Equation," *Soviet Physics Doklady,* Vol. 27, No. 1, Jan. 1982, pp. 29–31.

[18]Wang Chang, C. S., "On the Theory of the Thickness of Weak Shock Waves," *Studies in Statistical Mechanics,* edited by J. De Boer and G. E. Uhlenback, Vol. V, American Elseuler Publishing Co., New York, 1970, pp. 27–42.

[19]MacCormack, R. W., "Current Status of Numerical Solution of the Navier-Stokes Equations," AIAA Paper 85-0032, Jan. 1985.

[20]Zhong, X., MacCormack, R. W., and Chapman, D. R., "Stabilization of the Burnett Equations and Application to High-Altitude Hypersonic Flows," AIAA Paper 91-0770, Jan. 1991.

[21]Lumpkin, F. E., III, and Chapman, D. R., "Accuracy of the Burnett Equations for Hypersonic Real Gas Flows," AIAA Paper 91-0771, Jan. 1991.

[22]Tannehill, J. C., and Eisler, R. G., "Numerical Computation of the Hypersonic Leading Edge Problem Using the Burnett Equations," *The Physics of Fluids,* Vol. 19, No. 1, 1976, pp. 9–15.

[23]Schamberg, R., "The Fundamental Differential Equations and the Boundary Conditions for High Speed Slip Flow," Ph.D. Thesis, California Inst. of Technology, Pasadena, CA, 1947.

[24]Feiereisen, W. J., "The Hypersonic Double Ellipse in Rarefied Flow (Problem 6.4)," Inst. National de Recherche en Informatique et en Automatique Workshop on Hypersonic Flows for Reentry Problems, Antibes, France, Jan. 1990.